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SUMMARY

GRAND CHALLENGES

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This edition of the SPHERIC Newsletter is dedicated to the SPHERIC Grand Challenges (GCs). The GCs were initiated 5 years ago to bring the SPH community's attention to areas of SPH that prevent the more widespread development and use of SPH. Specifically, the issues highlighted by the GCs must be addressed for SPH to compete with more established methods, such as finite difference, finite volume, finite element, etc., whose theoretical foundations have been secured and whose state-of-the-art simulation packages are mature. It is essential that the SPH community around the world collaborates and addresses these GCs. Without being able to demonstrate characteristics and behaviour that are fundamental to any numerical method, SPH will continue to be considered as a "toy method" or "not serious". This is unacceptable. In the past decade, SPH has made massive progress, and this is evidenced by the increasing interest and uptake of the method, by developers and users in both industry and research. However, there is still much work to do. The GCs have therefore been formulated to focus the world-wide developmental efforts in taking SPH to the next level. The Monaghan prize, awarded in 2015 and 2018, has been instigated to highlight and reward outstanding work that helps address the GCs. Hence, the SPHERIC Steering Committee considered it timely to ask the Leaders or leading figures of each GC to summarise the current state-of-the-art in their respective challenge. You will find each GC described here in this newsletter. You are all strongly encouraged to focus your attention on helping this collaborative effort.

New SPHERIC forum dedicated to Grand Challenges: http://spheric-sph.org/forum/

Grand Challenge 1: convergence, consistency and stability

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The notions of convergence, consistency and stability are fundamental and underpin all numerical methods, with these concepts easier to formalise in some methods than others. SPH is a method where there remains a significant lack of understanding and formalism concerning all three and, quite rightly, addressing this is a Grand Challenge. I provide a personal viewpoint here, mentioning some recent works in the literature that shine more light on these issues in SPH, as well as posing a few philosophical questions to stir debate.

The above 3 properties are of course interlinked but are clearly distinct: a method may converge, but not be stable; it may be consistent to some level but not converge as expected. Regarding stability, we have always been quite lucky in SPH relative to other methods in being able to obtain physically meaningful results for time steps or resolutions where other methods often break down. Historically the pairing and tensile instabilities have been a concern but our understanding has much improved in recent years; for example, consider the pairing instability and the benefits of the Wendland kernels with non-negative Fourier transforms (Dehnen & Aly, 2012). Clearly, particle distribution is key to maintaining stability and additional numerical treatments that improve distributions, like particle shifting (Xu et al., 2009), have increased in popularity in recent years given their efficacy and relative ease to implement. Practically speaking stability can also be maintained through diffusion (physical or numerical), and following the earliest uses of artificial viscosity we now have some sophisticated approaches including, for example, deltaplus-SPH (Sun et al. 2017), combining diffusive terms in the conservation of mass equation with shifting for improved particle distributions. We are still a long way off formalising much of this – important headway is being made regarding stability in time stepping in weakly compressible SPH (Violeau and Leroy, 2014) and in incompressible SPH (Violeau and Leroy, 2015; Imoto 2018) – but a continued goal should be the determination of well-defined stability regions with bounds that have a known dependence on discretisation and kernel parameters, physical parameters, and numerical treatment parameters (e.g. shifting coefficients, delta parameters). The opportunity for further input from mathematicians/numerical analysts here is great.

Like stability, convergence depends critically on particle distributions. Quantification of the error during the smoothing operation is known. Numerical integration error can also be quantified when we split our integral into equi-spaced rectangles (particles) as per the rectangle or trapezoid rules. Over uniform (e.g. Cartesian) arrays of particles SPH can be shown to converge in numerical experiments with rates of convergence matching theoretical error measures extremely well. However, as soon as some level of particle disorder is introduced things becomes far more difficult. Errors and convergence rates are much more difficult to quantify, with convergence flat-lining, even diverging, once particles become sufficiently disordered – not ideal when your particles are Lagrangian. This close dependence of convergence on particle distribution seems to have motived a growing number of researchers to explore Arbitrary Lagrangian Eulerian (ALE) formulations of SPH. The fully Eulerian SPH method can converge readily and to high-orders of spatial accuracy (e.g. Lind and Stansby 2016; see Figure 1), while ALE-SPH (for example, Oger et al. 2016) permits study of a greater class of flows while also allowing control over particle distributions in order to improve accuracy and convergence. There is some really promising ongoing work here and this is an encouraging pathway – after all, even if one strongly values the Lagrangian nature of classical SPH, a legitimate question is whether the determined particle velocity is indeed the Lagrangian velocity. Of course, mathematical formalism is lacking here also, and quantification of

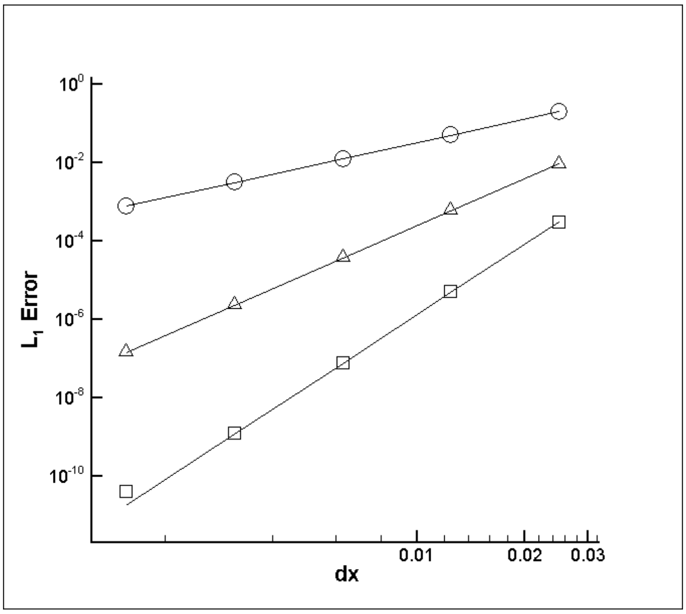

Figure 1 - High-order convergence of an SPH gradient for different kernels (see Lind & Stansby (2016) for more information).

error and convergence rates for irregular distributions in particular should be a key goal.

Consistency and convergence are closely linked and while consistent formulations may be constructed for arbitrary particle distributions, this can be costly and convergence is not necessarily ideal. Again, particle distributions remain key and recent investigations have focused on iterative redistribution, procedures based on transport velocities (Litvinov et al. 2015) or shifting (Vacondio and Rogers, 2017; Khayyer et al., 2019) that help to recover consistency without correction.

In summary, a key goal of this grand challenge remains in improving the mathematical formalism around quantification of error, convergence and stability, as well as enabling us to run informed simulations with confidence, this will inspire confidence in SPH in external fields and in industry. However, we should also not be afraid to question and to highlight nuance. For example, what do we mean by convergence? If we are solving a PDE, assuming there is a solution, then convergence becomes meaningful. If, however, we are working at the mesoscale, where many fashionable problems reside and where the continuum hypothesis starts to break down, the discrete particle system (that was always underlying) becomes apparent, and our usual notion of convergence loses meaning (i.e. we do not want Δx to go to 0!). But of course, it is in such examples of the versatility and flexibility of SPH that we find the reasons for its great appeal.

- Dehnen, W. and Aly, H., 2012. Improving convergence in smoothed particle hydrodynamics simulations without pairing instability. Monthly Notices of the Royal Astronomical Society, 425(2), pp.1068-1082.
- Xu, R., Stansby, P. and Laurence, D., 2009. Accuracy and stability in incompressible SPH (ISPH) based on the projection method and a new approach. Journal of computational Physics, 228(18), pp.6703-6725.
- Sun, P.N., Colagrossi, A., Marrone, S. and Zhang, A.M., 2017. The delta plus-SPH model: simple procedures for a further improvement of the SPH scheme. Computer Methods in Applied Mechanics and Engineering, 315, pp.25-49.
- Violeau, D. and Leroy, A., 2014. On the maximum time step in weakly compressible SPH. Journal of Computational Physics, 256, pp.388-415.
- Violeau, D. and Leroy, A., 2015. Optimal time step for incompressible SPH. Journal of Computational Physics, 288, pp.119-130.
- Imoto, Y., 2018. Unique solvability and stability analysis for incompressible smoothed particle hydrodynamics method. Computational Particle Mechanics, pp.1-13.
- Lind, S.J. and Stansby, P.K., 2016. High-order Eulerian incompressible smoothed particle hydrodynamics with transition to Lagrangian free-surface motion. Journal of Computational Physics, 326, pp.290-311.
- Oger, G., Marrone, S., Le Touzé, D. and De Leffe, M., 2016. SPH accuracy improvement through the combination of a quasi-Lagrangian shifting transport velocity and consistent ALE formalisms. Journal of Computational Physics, 313, pp.76-98.
- Litvinov, S., Hu, X.Y. and Adams, N.A., 2015. Towards consistence and convergence of conservative SPH approximations. Journal of Computational Physics, 301, pp.394-401.
- Vacondio, R., Rogers, B.D., 2017. Consistent Iterative shifting for SPH methods. Proceedings of the twelfth international SPHERIC workshop, Ourense, Spain (June 2017).
- Khayyer, A., Gotoh, H. and Shimizu, Y., 2019. A projection-based particle method with optimized particle shifting for multiphase flows with large density ratios and discontinuous density fields. Computers & Fluids, 179, pp.356-371.

Grand Challenge 1: Conservation and consistency of SPH method for incompressible flow simulation

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When solving incompressible fluid dynamics with SPH there are mainly two basic formulations, namely weakly compressible SPH (WCSPH) (Monaghan, 1994) and incompressible SPH (ISPH) (Cummins and Rudman, 1999). The former treats the fluid as weakly compressible with an equation of state that relates the fluid density to a hydrodynamic pressure. By imposing a Mach number around 0.1, the density fluctuations remain small and the fluid behaves quasi-incompressibly. In the latter formulation, similar to mesh-based methods, the incompressibility is enforced by solving a Poisson equation with a source term proportional to the velocity divergence or the density variation.

According to the theory of incompressible flow, the pressure field takes effect merely by its gradient. Therefore, the flow is invariant of a superimposed constant background pressure, i.e. it is gauge invariant. However, this is not the case for both the above mentioned formulations. The SPH simulation results can be highly dependent on the choice of the background pressure, and the method suffers from a dilemma of it. By analyzing the errors of SPH approximation, it is also found that this dilemma is actually rooted in another fundamental dilemma of conservation and consistency. To demonstrate the dilemma, we first consider the flow through a periodic lattice of cylinders as presented in Morris et al. (1997) where a cylinder is placed in a periodic box and the flow around the cylinder is driven by a body force in x-direction. The fluid is characterized by a Reynolds number of one. If this case is simulated with the classical WCSPH method without background pressure a void region occurs in the wake. This artifact is termed as tensile instability (Swegle et al. 1995). It occurs when the pressure becomes negative and results in artificial particle clumping or void regions. Such deficiency is leads to a similar behavior which is observed in molecule dynamics (MD) simulations, where the attractive forces lead to the molecule clustering in self-organized patterns. However, as SPH is a method computing macroscopic continuum dynamics, such microscopic clustering is a typical artifact. In order to eliminate this artifact it is necessary to use a suitably adjusted background pressure to ensure everywhere non-negative pressure in the entire domain.

We then consider the two-dimensional Taylor–Green flow at Re = 100. There is an analytical solution of the incompressible Navier–Stokes equation for this periodic array of vortices. At t = 0 we initialize the particle velocity with the analytical solution using a reference velocity of one. The numerical results, as shown in Fig. 1, indicate that when a background pressure is used, large extra numerical dissipation is introduced such that the decay rate is over predicted. This becomes very serious when the initial particles are on lattice positions. When they are initialized from the particle distribution at the end of previous simulations, the extra dissipation decreases, however, as shown in Fig. 1, it is still far beyond the acceptable range.

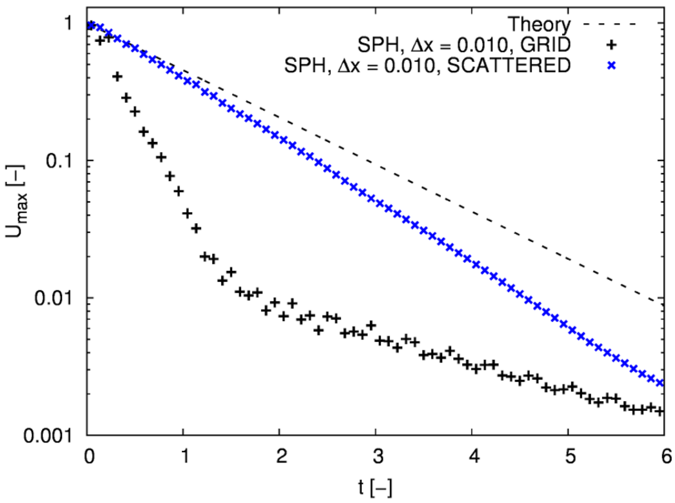

Figure 1 - Taylor-Green vortex at Re = 100. Decay of Umax with the initial particles at lattice (plus) and relaxed from previous simulation (cross).

This two examples leads to a dilemma of background pressure: if it is not applied the simulation is prone to tensile instability; if it is applied, the simulation produces unacceptable numerical dissipation. The tensile instability is due to the lack of zero-order consistency of the anti-symmetric SPH approximation, which leads to a dilemma of conservation and consistency.

The SPH discretization of derivatives has two typical formulations, i.e. the anti-symmetric and symmetric formulations. The numerical errors due to these two formulations are quite complex and are strongly dependent on particle distribution (Quinlan et al., 2006). With the anti-symmetric formulation, the SPH discretization for computing the pressure forces acting on a particle assumes the form implies momentum conservation of the particle system does not estimate correctly the vanishing gradient of a constant scalar field. This inconsistency suggests non-vanishing total force acting on a particle in a field with constant pressure. On the other hand, with the symmetric formulation, the SPH discretization for computing the density variation of a particle assumes the form zero-order consistency that a uniform velocity leads to vanishing of density variation. One may expect to cancel inconsistency error for the pressure field by applying the symmetric formulation to the discretized momentum equation. The dilemma is that the conservation of momentum, one of the most important properties of the original SPH method (Monaghan 1992, 2005), is not satisfied any more.

In some cases, the background dilemma is not obvious. One example is free surface flow problems for which SPH often out-performs other traditional CFD methods. This is because in this case the pressure generally is kept positive due to the gravity and the technique by which the zero pressure is assumed at the free surface. Another example is the astrophysical simulations in which the equation of state for compressible fluid is applied. Since the thermodynamic pressure never go negative, the tensile instability seldom shows up.

The main approaches recently developed addressing this conservation and consistency of SPH rely on slightly modified particle advection velocity. The idea of moving particles with a transport velocity which may differ from the momentum velocity was first proposed with the XSPH scheme to prevent penetration in impact problems (Monaghan 1989). Hu and Adams (2007, 2009) utilized the transport velocity obtained from an intermediate projection step to impose fluid incompressibility. Xu et al. (2009) developed a shifting approach in the incompressible SPH method for homogenized particle distribution. Lind et al. (2012) extended this approach to simulate free surface flow with a surface-identification algorithm. Vacondio et al. (2013) modified this approach for a variable-resolution SPH method. Monaghan (2011) developed an SPH turbulence model in which the used smoothed transport velocity can be related to the Lagrangian averaged Navier-Stokes equations (LANS) (Holm 1999). Adami et al. (2013) proposed a transport-velocity formulation to address particle clumping and void-region problems in weakly-compressible SPH simulation of flow at high Reynolds number. Using a globally constant background pressure for regulation, the transport velocity leads to favorable particle distribution and reduces numerical error (Litvinov et al. 2015). A very recent development is that variable background pressure is introduced to address solid dynamics problem and fluid problems with free surface flow (Zhang et al. 2017).

- Monaghan, J. J. Simulating free surface flows with SPH. J. Comput. Phys., , 1994, 110, 399-406
- Cummins, S. J. & Rudman, M. An SPH projection method. J. Compu. Phys., 1999, 152, 584
- Morris, J. P.; Fox, P. J. & Zhu, Y. Modeling low Reynolds number incompressible flows using SPH. J. Comput. Phys., 1997, 136, 214
- Swegle, J. W.; Hicks, D. L. & Attaway, S. W. Smoothed Particle Hydrodynamics stability analysis. J. Comput. Phys. , 1995, 116, 123-134
- Quinlan, N. J.; Basa, M. & Lastiwka, M. Truncation error in mesh-free particle methods. Int. J. Numer. Methods. Eng., 2006, 66, 2064-2085
- Monaghan, J. Smoothed particle hydrodynamics. Ann. Rev. Astronom. Astrophys., 1992, 30, 543
- Monaghan, J. J. Smoothed Harticle Hydrodynamics. Reports on progress in physics, IOP Publishing, 2005, 68, 1703
- Monaghan, J. On the problem of penetration in particle methods. J. Comput. Phys., , 1989, 82, 1-15
- Hu, X. Y. & Adams, N. A. An incompressible multi-phase SPH method. J. Comput. Phys., 2007, 227, 264-278
- Hu, X. & Adams, N. A constant-density approach for incompressible multi-phase SPH J. Comput. Phys., 2009, 228, 2082-2091
- Xu, R.; Stansby, P. & Laurence, D. Accuracy and stability in incompressible SPH (ISPH) based on the projection method and a new approach. J. Comput. Phys., , 2009, 228, 6703-6725
- Lind, S.; Xu, R.; Stansby, P. & Rogers, B. D. Incompressible smoothed particle hydrodynamics for free-surface flows: A generalised diffusion-based algorithm for stability and validations for impulsive flows and propagating waves. J. Comput. Phys., 2012, 231, 1499-1523
- Vacondio, R.; Rogers, B.; Stansby, P.; Mignosa, P. & Feldman, J. Variable resolution for SPH: a dynamic particle coalescing and splitting scheme. Computer Methods in Applied Mechanics and Engineering, 2013, 256, 132-148
- Monaghan, J. A turbulence model for Smoothed Particle Hydrodynamics European. Journal of Mechanics-B/Fluids, 2011, 30, 360-370
- Holm, D. D. Fluctuation effects on 3D Lagrangian mean and Eulerian mean fluid motion. Physica D, 1999, 133, 215-269
- Adami, S.; Hu, X. & Adams, N. A transport-velocity formulation for smoothed particle hydrodynamics. J. Comput. Phys., 2013, 241, 292-307
- Litvinov, S.; Hu, X. Y. & Adams, N. A. Towards consistence and convergence of conservative SPH approximations. J. Comput. Phys., 2015, 301, 394-401
- Zhang, C.; Hu, X. Y. & Adams, N. A. A generalized transport-velocity formulation for smoothed particle hydrodynamics. J. Comput. Phys., 2017, 337, 216-232

Grand Challenge 2: Boundary Conditions

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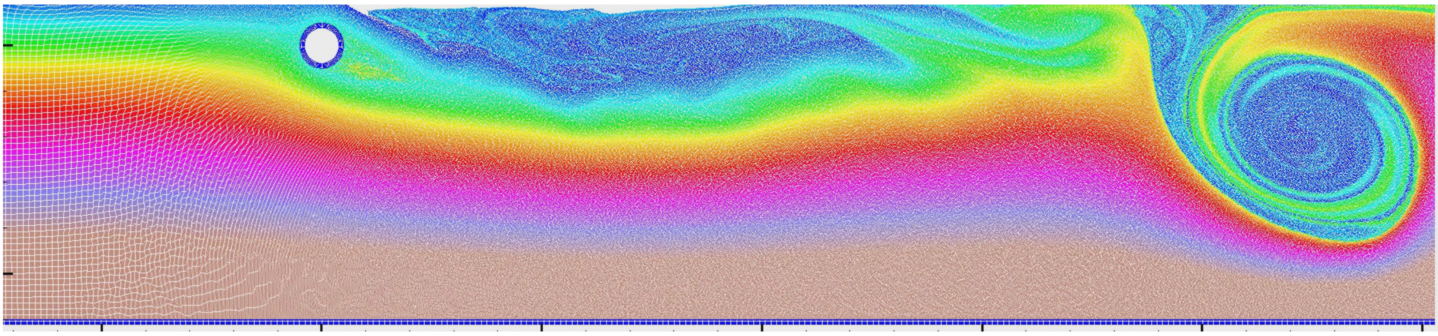

Figure 1 Flow modeled with SPH making evident the need for back-flow modeling (from Bouscasse et al. 2017).

In order to close the fluid dynamics equations, initial (ICs) and boundary conditions (BCs) are necessary. BC include solid boundaries (free slip, no slip, pressure normal derivative), free surface, inlet/oulet, (aka open BCs - OBCs), stress conditions in structural mechanics, those related to the coupling with other models, etc., ICs are included in this challenge since they usually require special treatment in SPH, e.g. when a hydrostatic condition is needed. This need arises mostly due to unfeasibility to exactly link mass and volume in SPH.

The purpose of this short communication is to review some recent literature and identify topics worthy of further research. It is relevant to mention that in recent SPH review papers (Gomez-Gesteira et al., 2010, Gotoh and Khayyer, 2016, Monaghan, 2005, 2012, Violeau and Rogers, 2016), there are specific review sections on BCs. The influential review by Price (2012) does not however contain any reference to BCs as, in Astrophysics, they are less of an issue than in typical Engineering scales.

To include ICs and BCs in SPH, researchers use various techniques. There are a number of key issues that remain to be fully addressed, such as:

1. How to include BCs without loosing intrinsic SPH conservation properties?
2. How to include BCs consistently and without compromising stability? This is directly related with the role of boundary integrals.
3. How to include solid wall BCs for actual geometries with complex shapes (2D, 3D)?
4. How to provide an initial distribution of particles which avoids the onset of shocks once the time-integration starts?
5. How to treat contact lines between free-surfaces and solid boundaries?
6. How to treat back-flows (aka recirculation) when implementing OBCs?
7. How to implement BCs in the interface between subdomains solved with different methods?
8. How to accurately impose BCs in Incompressible SPH (ISPH) in complex flows?
9. How to accurately impose BCs when particle shifting (within a consistent ALE framework or not) is used?

Some recent interesting references have looked into these issues: Ni et al. (2018) implemented a wave flume with SPH using OBCs but did not look into recirculation issues. Along the same line, Bouscasse et al. (2017) used OBCs for simulating the viscous flow around a submerged cylinder. In order to avoid back-flow, they had to significally extend the flow domain upstream and downstream, as well as limiting the simulation time (see Fig. 1). Back flow is held in FVM-VOF methods by indicating the physical properties of the incoming fluid, applying to it the local flow properties (velocity, temperature, etc.) but it is not clear how to implement it a Lagrangian approach. Tafuni et al. (2018) have recently extended OBC algorithms to the popular GPU HPC implementation DualSphysics, and Wang et al (2019) have proposed a novel OBC implmenteation based on the method of characteristics using timeline interpolations.

Long-time simulations of free-surface flows have been traditionally an issue in SPH due to the onset of stability problems. However Green and Peiró (2018) have recently been able to carry out long and accurate simulations of flows inside tanks by using fixed/prescribed motion dummy particles developped by Adami et al. (2012), and by performing a good selection of simulation parameters. Regarding BCs affecting consistency of the operators, Fougeron and Aubry (2019) have proposed a novel method based on non-boundary fitted clouds of points; they redefine the Lagrangian nature of the model by creating a set of nodes on the boundary, which then use to approximate the differential operators. They use this approach in elliptic equations and though appealing ideas can be found, the application to typical SPH problems, such as wave-body interactions, is not evident to us. Regarding energy conservation and BCs, Cercos-Pita et al. (2017) investigated the energy conservation properties of SPH in the presence of fluid–solid interactions. They showed that due to the solid BC, the energy equation of the particle system contains some extra terms that tend to vanish when the spatial resolution is increased (very slowly), and that affect the energy conservation of the system. Based on the test cases they run, they conjecture that the contribution is dissipative, but no rigorous proof is provided. Regarding ISPH, Takahashi et al. (2018) provided an interesting discussion on the difficulties of imposing Dirichlet and Neumann BCs, including some improvements. Finally, regarding ALE formulations, Oger et al. (2016) reported the need to remove shifting when close to the free surface, defining in turn the ghost fluid properties without requring any specific ALE related correction.

- Adami, S., Hu, X. Y., and Adams, N. A. (2012). A generalized wall boundary condition for smoothed particle hydrodynamics. Journal of Computational Physics, 231(21):7057-7075.
- Bouscasse, B., Colagrossi, A., Marrone, S., and Souto-Iglesias, A. (2017). SPH modelling of viscous flow past a circular cylinder interacting with a free surface. Computers & Fluids, 146:190-212.
- Cercos-Pita, J. L., Antuono, M., Colagrossi, A., and Souto-Iglesias, A. (2017). SPH energy conservation for fluid–solid interactions. Computer Methods in Applied Mechanics and Engineering, 317:771-791.
- Fougeron, G. and Aubry, D. (2019). Imposition of boundary conditions for elliptic equations in the context of non boundary fitted meshless methods. Computer Methods in Applied Mechanics and Engineering, 343:506-529.
- Gomez-Gesteira, M., Rogers, B. D., Dalrymple, R. A., and Crespo, A. J. C. (2010). State-of-the-art of classical SPH for free-surface flows. Journal of Hydraulic Research, 48(S1):6-27.
- Gotoh, H. and Khayyer, A. (2016). Current achievements and future perspectives for projection-based particle methods with applications in ocean engineering. Journal of Ocean Engineering and Marine Energy, 2(3):251-278.
- Green, M. D. and Peiró, J. (2018). Long duration SPH simulations of sloshing in tanks with a low fill ratio and high stretching. Computers & Fluids, 174:179-199.
- Monaghan, Smoothed particle hydrodynamics. Reports on Progress in Physics, 68:1703–1759, 2005.
- Monaghan, J.J., Smoothed particle hydrodynamics and its diverse applications. Annual Review of Fluid Mechanic, 44(1):323–346, 2012.
- Ni, X., Feng, W., Huang, S., Zhang, Y., and Feng, X. (2018). A SPH numerical wave flume with non-reflective open boundary conditions. Ocean Engineering, 163:483-501.
- Oger, G., Marrone, S., Le Touzé, D., and de Leffe, M. (2016). SPH accuracy improvement through the combination of a quasi-Lagrangian shifting transport velocity and consistent ALE formalisms. Journal of Computational Physics, 313:76-98.
- Price, D.J. (2012). Smoothed particle hydrodynamics and magnetohydrodynamics. Journal of Computational Physics, 231(3):759-794.
- Tafuni, A., Domínguez, J. M., Vacondio, R., and Crespo, A. J. C. (2018). A versatile algorithm for the treatment of open boundary conditions in smoothed particle hydrodynamics GPU models. Computer Methods in Applied Mechanics and Engineering, 342:604-624.
- Takahashi, T., Dobashi, Y., Nishita, T., and Lin, M. (2018). An efficient hybrid incompressible SPH solver with interface handling for boundary conditions. Computer Graphics Forum, 37(1):313-324.
- Violeau, D. and Rogers, B. D. (2016). Smoothed particle hydrodynamics (SPH) for free-surface flows: Past, present and future. Journal of Hydraulic Research, 54(1):1-26.
- Wang, P., Zhang, A.-M., Ming, F., Sun, P., and Cheng, H. (2019). A novel non-reflecting boundary condition for fluid dynamics solved by smoothed particle hydrodynamics. Journal of Fluid Mechanics, 860:81-114.

Grand Challenge 3: Adaptivity

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Adaptivity is the capability of a numerical scheme o use a domain discretization based on elements with different size. For Eulerian mesh-based methods such as Finite Volume, Finite Elements or Finite Differences those elements are the grid cells, whereas in Lagrangian meshless-based numerical methods they are the computational nodes that moves with the fluid velocity.

Adaptivity is a crucial feature for numerical schemes. It allows us to increase the number of computational nodes (cells or particles) only in the portions of the domain where the flow features require higher resolution. In this way the total number of computational nodes (and so the computational cost for the simulation) used to discretize a domain can be dramatically decreased, for a given level of error.

In mesh-based methods, variable resolution is a common feature and it has been introduced in several different ways. Often referred to as Adaptive Mesh Refinement (AMR) the most common approaches are unstructured grids or quadtree grids. Moreover, several different algorithms have been used successfully to dynamically adjust the mesh resolution, accordingly to some measures of the discretization error or smoothness indicators for the numerical solutions (see for example Dumbser et al. 2013, Johnson et al. 1992).

Despite the need to introduce variable resolution in SPH numerical schemes for fluids, almost all SPH codes are based on uniform resolution and this prevents the use of SPH models to simulate all Engineering problems which are inherently multiscale. For compressible fluids and astrophysical simulations, variable resolution has been introduced many years ago in SPH models by varying the size of the kernel accordingly to the density field, and ensuring the conservation of fundamental properties by deriving the formulation using an Hamiltonian approach (Gingold and Monaghan, 1982; Hernquist and Katz, 1989). Unfortunately, the same approach cannot be used for weakly compressible (or strictly incompressible) fluids where density remains (approximately) constant.

Initial efforts have been made for weakly compressible SPH models by introducing regions with different resolution at the beginning of the simulations (Bonet and Rodríguez-Paz 2005, Børve et al. 2005, Oger et al. 2006, Omidvar et al. 2012, Omidvar et al. 2013).

Afterwards, with the aim of dynamically varying the particle resolution, some authors proposed some procedures to dynamically increase and reduce the particle resolution (Barcarolo, et al., 2014; Reyes López et al. 2013; Vacondio et al. 2013; Vacondio et al. 2016).

Very recently, Sun et al. (2018) simulated flow past different bodies in presence of a free surface by using the Adaptive Particle Refinement (APR) methodology proposed by Chiron et al. (2018). Spreng et al. (2019) proposed a criterion to automatically adjust the particle resolution accordingly to some measure of the SPH spatial discretization error.

Despite the progresses in developing dynamic particle adaptivity we think that some major challenges have still to be addressed in order to obtain a methodology that is robust enough to be adopted by practitioners and industry.

Looking far into the future, from the users' perspective, dynamic adaptivity should be fully automated and activated only when needed. Full automation requires criteria to be developed that control the activation. A question then arises as to what these criteria should be and how they should operate? While this has been well investigated in adaptive mesh refinement (AMR), the same concepts do not necessarily apply in SPH since the nature of the discretisation is different. Most importantly, it is presently unclear what is the best general approach, and this requires (i) a focused research effort from the SPH community and (ii) an understanding from users that implementing and using adaptivity in SPH faces some key challenges and is far from straightforward. However, it is already clear that there are at least 3 key objectives:

(i) Error minimisation: it is impossible to avoid the introduction of error, but any form of SPH adaptivity should guarantee that the error has been minimised. To date, limited attention has been given to this (Feldman et al. 2007, Vacondio et al. 2013, 2016). Too often, schemes simply split particles into an arbitrary number (for example 4) of so-called daughter particles (motivated by simplicity or ease-of-coding) with little consideration of the error and how it propagates throughout the solution. Similar to mature AMR schemes, error minimisation is a natural candidate as a criterion to for APR.

(ii) Uniform error distribution: Ideally, the error in an SPH simulation should still be uniform across the domain. There should be no sacrifice in error solely due to the requirement to use dynamic adaptation of particles in specific regions.

(iii) Robust schemes for all applications: due to its flexibility, the range of SPH applications is huge with highly complex processes. This naturally presents a challenging question – how to develop particle adaptivity that is widely applicable and robust? If certain types of adaptivity only work for a restricted number or type of applications, this calls into question the validity of the approach – in practice this means ensuring consistency and convergence.
In addition to the theoretical considerations and developments, there are multiple challenges going forward:
- Implementation with HPC and emerging technology: Even with APR, with its discretisation SPH will need some form of hardware acceleration for the foreseeable future. In the past decade there has been a fundamental shift from faster clock speeds to different types of parallelism. For adaptivity, this poses the challenge of implementation. With different types of hardware continually appearing, developing implementations of adaptivity that are future-proofed will avoid costly recoding.
- Multi-phase implementations: Applications involving multiple phases can be extraordinarily complex, and to date, only simple cases or applications have been simulated in SPH. Developing robust adaptivity schemes for multi-phase flows whose properties can evolve represents a formidable challenge.

- D.A. Barcarolo, D. Le Touzé, G. Oger, F. de Vuyst, Adaptive particle refinement and derefinement applied to the smoothed particle hydrodynamics method, Journal of Computational Physics, vol. 273, 2014, pp. 640-657 doi: 10.1016/j.jcp.2014.05.040.
- J. Bonet, M.X. Rodríguez-Paz (2005), Hamiltonian formulation of the variable-h SPH equations, Journal of Computational Physics, 209 (2) pp. 541-558 doi: 10.1016/j.jcp.2005.03.030
- S. Børve, M. Omang, J. Trulsen, Regularized smoothed particle hydrodynamics with improved multi-resolution handling, Journal of Computational Physics, 208 (1), pp. 345–367 doi: 10.1016/j.jcp.2005.02.018
- L. Chiron, G. Oger, M. de Leffe, D. Le Touzé, Analysis and improvements of Adaptive Particle Refinement (APR) through CPU time, accuracy and robustness considerations, Journal of Computational Physics, vol. 354, 2018, pp. 552–575, doi: 10.1016/j.jcp.2017.10.041.
- M. Dumbser, O. Zanotti, A. Hidalgo, D. S. Balsara, ADER-WENO finite volume schemes with space–time adaptive mesh refinement, Journal of Computational Physics, vol. 248, 2013, pp. 257–286, doi: 10.1016/j.jcp.2013.04.017.
- R.A. Gingold and J.J. Monaghan, Kernel Estimates as a Basis for General Particle Methods in Hydrodynamics, Journal of Computational Physics, vol. 46, no. 3, 1982, pp. 429–453, doi:10.1016/0021-9991(82)90025-0.
- L. Hernquist and N. Katz, TREESPH - A Unification of SPH with the Hierarchical Tree Method, The Astrophysical Journal Supplement Series, vol. 70, 1989, p. 419-446, doi:10.1086/191344.
- C. Johnson and P. Hansbo, Adaptive Finite Element Methods in Computational Mechanics, Computer Methods in Applied Mechanics and Engineering, vol. 101, no. 1-3, 1992, pp. 143–181, doi:10.1016/0045-7825(92)90020-k.
- Y.R. López, D. Roose, C. R. Morfa, Dynamic Particle Refinement in SPH: Application to Free Surface Flow and Non-Cohesive Soil Simulations, Computational Mechanics, vol. 51, no. 5, 2012, pp. 731–741, doi: 10.1007/s00466-012-0748-0.
G. Oger, M. Doring, B. Alessandrini, P. Ferrant, Two-dimensional SPH simulations of wedge water entries, Journal of Computational Physics, vol. 213, no. 2, 2006, pp. 803–822, doi: 10.1016/j.jcp.2005.09.004.
- P. Omidvar, P.K. Stansby, B.D. Rogers, SPH for 3D Floating Bodies Using Variable Mass Particle Distribution, International Journal for Numerical Methods in Fluids, vol. 72, no. 4, 2012, pp. 427–452, doi: 10.1002/fld.3749.
- P. Omidvar, P.K. Stansby, B.D. Rogers, Wave Body Interaction in 2D Using Smoothed Particle Hydrodynamics (SPH) with Variable Particle Mass, International Journal for Numerical Methods in Fluids, vol. 68, no. 6, 2011, pp. 686–705, doi: 10.1002/fld.2528.
- P.N. Sun, A. Colagrossi, S. Marrone, M. Antuono, A.M. Zhang Multi-Resolution Delta-plus-SPH with Tensile Instability Control: Towards High Reynolds Number Flows, Computer Physics Communications, vol. 224, 2018, pp. 63–80, doi: 10.1016/j.cpc.2017.11.016.
- F. Spreng, R. Vacondio, P. Eberhard, J.R. Williams, An Advanced Study on Discretization-Error-Based Adaptivity in Smoothed Particle Hydrodynamics, Computers & Fluids, Forthcoming.
- R. Vacondio, B.D. Rogers, P.K. Stansby, P. Mignosa, Variable Resolution for SPH in Three Dimensions: Towards Optimal Splitting and Coalescing for Dynamic Adaptivity, Computer Methods in Applied Mechanics and Engineering, vol. 300, 2016, pp. 442–460, doi: 10.1016/j.cma.2015.11.021.
- R. Vacondio, B.D. Rogers, P.K. Stansby, P. Mignosa, J. Feldman, Variable Resolution for SPH: A Dynamic Particle Coalescing and Splitting Scheme, Computer Methods in Applied Mechanics and Engineering, vol. 256, 2013, pp. 132–148, doi: 10.1016/j.cma.2012.12.014.

Grand Challenge 4: Coupling to other models

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The SPH method is naturally able to resolve multi-mechanics problems and include different physical models in its meshless formalism. As other Lagrangian meshless methods, SPH is very accurate and efficient when dealing with moving boundaries and complex interfaces, which are generally addressed with difficulties by conventional numerical methods (e.g., FVM, FEM). However, for problems where the latter methods are currently used and well established the SPH is generally less effective and, for the same level of attained accuracy, results more costly. In several contexts, it can be much more effective to couple an SPH solver to another numerical solver, thus enhancing the capabilities of both methods within their specific application fields. In this way, a wider range of problems is efficiently addressed.
The coupling algorithm and the related implementation complexity can largely vary depending on several aspects:
- One-way (offline) or two-way coupling;
- Heterogeneity of the modelled physics (e.g. potential flow/Navier-Stokes, fluid/solid, compressible/incompressible, etc.);
- Lagrangian or Eulerian approach adopted in the method coupled to SPH;
- Discrete coupling interfaces between solvers (mesh/meshless, sharp interface/blending region, etc.);
- Time stepping and stability of the coupled algorithm (e.g. explicit/implicit time integration, multiple time step);
- Preservation of conservative quantities by the coupling.
Besides, the complexities related to the coupling of very different solvers can be counterbalanced by impressive gains in terms of efficiency (see, e.g., Chiron et al. 2018).
Most of the works regarding SPH coupling address Fluid Structure Interaction (FSI) problems for which the solid structure is generally solved by Finite Element Methods (FEM) and Discrete Element Methods (DEM). The Lagrangian character of those model has allowed a quite fast development of this kind of coupling and has been targeted in the first attempts of coupling the SPH method (see Attaway et al. 1994). In particular, SPH-FEM coupling has reached a good maturity and has been used in several recent works addressing hydro-elasticity problems (see, e.g., Li et al. 2015, Yang et al. 2016, Long et al. 2016, Fourey et al. 2017) proving that this coupling paradigm can be highly competitive in FSI problems as in Siemann and Langrand (2017).
SPH-DEM coupling has been mostly used for problems

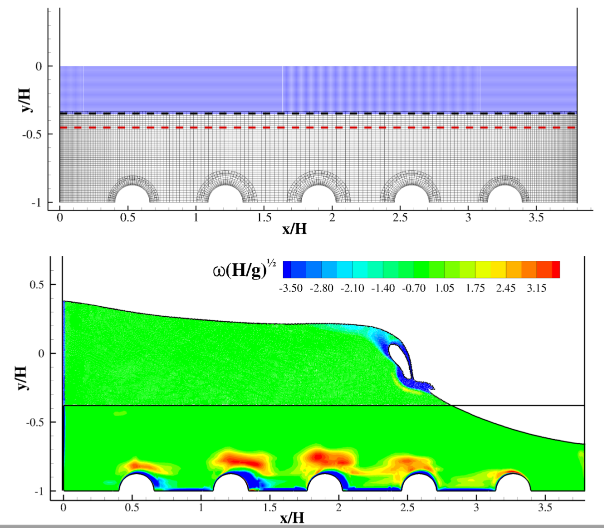

Figure 1 - Coupled SPH-FVM simulation of a sloshing flow in a tank with a corrugated bottom from Chiron et al. (2018). Top: SPH particles (blu) and FVM grid (black). Bottom: a time instant of the evolution showing vorticity contours and the free surface profile crossing the coupling interface.

in which several solid rigid bodies interact with a fluid flow (see e.g. Canelas et al. 2016, Robb et al. 2016) including granular flows (see Canelas et al. 2017, Markauskas et al 2017). Very recently coupling with open source multi-mechanics libraries has been implemented to simulate fluid-mechanism interactions by modelling frictional and multi-restriction based behaviors (see e.g. Canelas et al 2018).
Furthermore, SPH coupling has been largely develo-

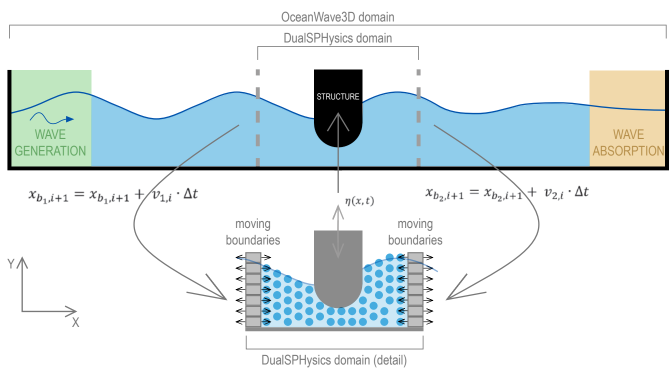

Figure 2 - Principle of 2D coupling between OceanWave3D and DualSPHysics around a structure under wave action from Verbrugghe et al. (2018). The top part shows the complete domain in OceanWave3D. The bottom part illustrates the DualSPHysics zone

ped for coastal engineering purposes. In this case SPH is coupled with NLSWe models (e.g. Altomare et al. 2016, 2018) or potential flow solvers in the form of Spectral Methods (e.g. Oger et al. 2014) or Finite Difference (Verbrugghe et al . 2018) for solving wave propagation in the far field and restraining SPH in the region where wave-structure interactions and wave-breaking are expected. This includes also the simulation of ship motions and the associated sloshing dynamics in the internal tanks as recently done in Serván-Camas et al. (2018) and Bulian and Cercos-Pita (2018).

Finally, a recent and growing branch is the coupling between Finite Volume Schemes (FVM) and SPH. In this case the coupling strategy aims at flow simulations in which the accuracy and the ability of grid stretching of the FVM can be usefully coupled with the SPH properties in modelling complex interfaces (see e.g. Marrone et al. 2016, Napoli et al. 2016, Fernandez-Gutierrez et al. 2018, Kumar et al. 2018).

Chiron, L., Marrone, S., Di Mascio, A., & Le Touzé, D. (2018). Coupled SPH–FV method with net vorticity and mass transfer. Journal of Computational Physics, 364, 111-136.
Attaway, S. W., Heinstein, M. W., & Swegle, J. W. (1994). Coupling of smooth particle hydrodynamics with the finite element method. Nuclear engineering and design, 150(2-3), 199-205.
Li, Z., Leduc, J., Nunez-Ramirez, J., Combescure, A., & Marongiu, J. C. (2015). A non-intrusive partitioned approach to couple smoothed particle hydrodynamics and finite element methods for transient fluid-structure interaction problems with large interface motion. Computational Mechanics, 55(4), 697-718.
Yang, X., Liu, M., Peng, S., & Huang, C. (2016). Numerical modeling of dam-break flow impacting on flexible structures using an improved SPH–EBG method. Coastal Engineering, 108, 56-64.
Long, T., Hu, D., Yang, G., & Wan, D. (2016). A particle-element contact algorithm incorporated into the coupling methods of FEM-ISPH and FEM-WCSPH for FSI problems. Ocean Engineering, 123, 154-163.
Fourey, G., Hermange, C. L. T. D., Le Touzé, D., & Oger, G. (2017). An efficient FSI coupling strategy between Smoothed Particle Hydrodynamics and Finite Element methods. Computer Physics Communications, 217, 66-81.
Siemann, M. H., & Langrand, B. (2017). Coupled fluid-structure computational methods for aircraft ditching simulations: Comparison of ALE-FE and SPH-FE approaches. Computers & Structures, 188, 95-108.
Ren, B., Jin, Z., Gao, R., Wang, Y. X., & Xu, Z. L. (2013). SPH-DEM modeling of the hydraulic stability of 2D blocks on a slope. Journal of Waterway, Port, Coastal, and Ocean Engineering, 140(6), 04014022.
Canelas, R. B., Crespo, A. J., Domínguez, J. M., Ferreira, R. M., & Gómez-Gesteira, M. (2016). SPH–DCDEM model for arbitrary geometries in free surface solid–fluid flows. Computer Physics Communications, 202, 131-140.
Canelas, R. B., Domínguez, J. M., Crespo, A. J. C., Gómez-Gesteira, M., & Ferreira, R. M. L. (2017). Resolved simulation of a granular-fluid flow with a coupled SPH-DCDEM model. Journal of Hydraulic Engineering, 143(9), 06017012.
Canelas, R.B., Brito, M., Feal, O.G., Domínguez, J.M., Crespo, A.J.C., 2018. Extending DualSPHysics with a Differential Variational Inequality: modeling fluid-mechanism interaction, Applied Ocean Research, Volume 76, 2018, 88-97.
Markauskas, D., Kruggel-Emden, H., Sivanesapillai, R., & Steeb, H. (2017). Comparative study on mesh-based and mesh-less coupled CFD-DEM methods to model particle-laden flow. Powder Technology, 305, 78-88.
Robinson, M., Ramaioli, M., & Luding, S. (2014). Fluid–particle flow simulations using two-way-coupled mesoscale SPH–DEM and validation. International journal of multiphase flow, 59, 121-134.
Altomare, C., Domínguez, J. M., Crespo, A. J. C., Suzuki, T., Caceres, I., & Gómez-Gesteira, M. (2016). Hybridization of the wave propagation model SWASH and the meshfree particle method SPH for real coastal applications. Coastal Engineering Journal, 57(4), 1550024-1.
Altomare, C., Tagliafierro, B., Dominguez, J. M., Suzuki, T., & Viccione, G. (2018). Improved relaxation zone method in SPH-based model for coastal engineering applications. Applied Ocean Research, 81, 15-33.
Oger, G., Le Touzé, D., Ducrozet, G., Candelier, J., & Guilcher, P. M. (2014). A Coupled SPH-Spectral Method for the Simulation of Wave Train Impacts on a FPSO. In ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering (pp. V002T08A088-V002T08A088). American Society of Mechanical Engineers.
Verbrugghe, T., Domínguez, J. M., Crespo, A. J., Altomare, C., Stratigaki, V., Troch, P., & Kortenhaus, A. (2018). Coupling methodology for smoothed particle hydrodynamics modelling of non-linear wave-structure interactions. Coastal Engineering, 138, 184-198.
Serván-Camas, B., Cercos-Pita, J. L., Colom-Cobb, J., García-Espinosa, J., & Souto-Iglesias, A. (2016). Time domain simulation of coupled sloshing–seakeeping problems by SPH–FEM coupling. Ocean Engineering, 123, 383-396.
Bulian, G., & Cercos-Pita, J. L. (2018). Co-simulation of ship motions and sloshing in tanks. Ocean Engineering, 152, 353-376.
Marrone, S., Di Mascio, A., & Le Touzé, D. (2016). Coupling of Smoothed Particle Hydrodynamics with Finite Volume method for free-surface flows. Journal of Computational Physics, 310, 161-180.
Napoli, E., De Marchis, M., Gianguzzi, C., Milici, B., & Monteleone, A. (2016). A coupled finite volume–smoothed particle hydrodynamics method for incompressible flows. Computer Methods in Applied Mechanics and Engineering, 310, 674-693.
Fernandez-Gutierrez, D., Souto-Iglesias, A., & Zohdi, T. I. (2018). A hybrid Lagrangian Voronoi–SPH scheme. Computational Particle Mechanics, 5(3), 345-354.
Kumar, P., Yang, Q., Jones, V., & McCue-Weil, L. (2015). Coupled SPH-FVM simulation within the OpenFOAM framework. Procedia IUTAM, 18, 76-84.

Grand Challenge 5: Applicability to industry

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Applicability of the SPH method to industry is related to several ingredients which are briefly described on the SPHERIC website. In the recent period, various contributions from the research community have brought significant progress likely to foster the adoption of SPH among industry.

Appearance of tools with Graphical User Interfaces for the pre- and post-processing of SPH simulations is noticeable. DesignSPHysics (Vieira et al., 2017) and VisualSPHysics (García-Feal et al., 2016) provide a complete simulation tool chain dedicated to SPH simulations. An alternative has been developed based on ParaView (Sur et al., 2017). Advanced analysis of flow features still rely mainly on the projection of the particles data onto a grid. For the creation of the initial particle distribution in complex geometries, the Particle Packing Algorithm has gained popularity as in (Dauch et al., 2017).

The ease of use of the method will probably benefit from the recent improvements of the dynamic and adaptive particle refinement techniques. Significant contributions in this field have been given by Vacondio et al. (2016) and Chiron et al. (2018). A further development of these techniques will relieve simulation engineers from the setting of the adequate particle size for their application cases.

A major challenge for the application in industry remains the tradeoff between computational cost and accuracy. In this field, emergence of incompressible modelling in SPH is certainly a salient point, especially in applications with moderate dynamics and / or a little partition of the free surface. Advances include improved robustness and accuracy of the method, coupled with improved numerics that manage to compensate the increased computational cost with a significantly bigger time step size. The GPU implementation of ISPH, as reported in (Chow et al., 2018), will probably reinforce its efficacy. The computation time of SPH simulations will also benefit from advances in the coupling of SPH with other numerical methods, allowing concentrating the SPH on the solving of specific flow regions only. An exemplary coupling of SPH and Finite Volumes can be found in (Chiron et al., 2018).

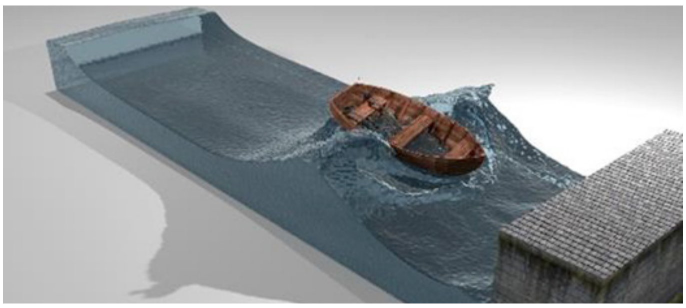


Figure 1 - Picture of floating boat done with VisualSPHysics. From Garcia-Feal et al., 2016

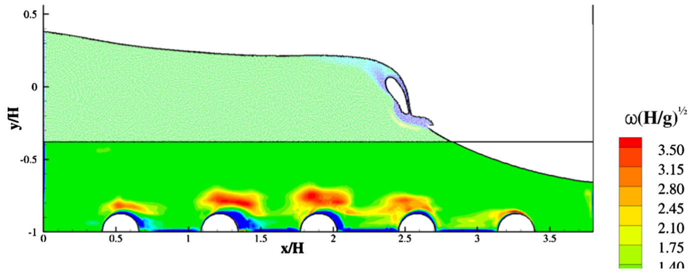
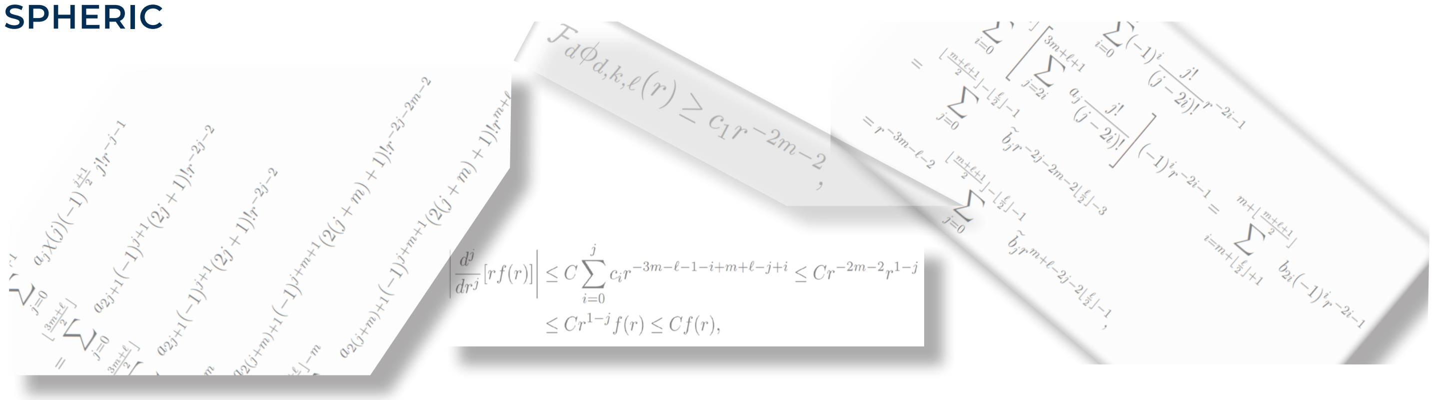


Figure 2 - Sloshing flow in a tank with a corrugated bottom – coupled solution. From Chiron et al., 2018

- Dauch T. F., Okraschevski M., Keller M. C., Braun S., Wieth L., Chaussonnet G., Kock R., «Preprocessing Workflow for the Initialization of SPH Predictions based on Arbitrary CAD Models», Proceedings of the 12th International SPHERIC workshop, Ourense, Spain, 2017.
- Sur C., van Beest B., "SPHStudio: A ParaView based software to develop SPH simulation models", Proceedings of the 12th International SPHERIC workshop, Ourense, Spain, 2017.
- Vieira A., Garcia-Feal O., Dominguez J.M., Crespo A.J.C., Gomez-Gesteira M., "Graphical User Interface for SPH codes: DesignSPHysics", Proceedings of the 12th International SPHERIC workshop, Ourense, Spain, 2017.
- O. García-Feal, A.J.C. Crespo, J.M. Domínguez, M. Gómez-Gesteira, "Advanced fluid visualization with DualSPHysics and Blender", Proceedings of the 11th International SPHERIC workshop, Munich, Germany, 2016.
- Vacondio R., Rogers B.D., Stansby P., Mignosa P., "Variable resolution for SPH in three dimensions: Towrads optimal splitting and coalescing for dynamic adaptivity", Computer Methods in Applied Mechanics and Engineering, 300: 442-460, April 2016. doi:10.1016/j.cma.2015.11.021
- Chow A.D., Rogers B.D., Lind S.J., Stansby P.K., "Incompressible SPH (ISPH) with fast Poisson solver on a GPU", Computer Physics Communications, 226: 81-103, 2018. doi: 10.1016/j.cpc.2018.01.005
L. Chiron, G. Oger, M. de Leffe, D. Le Touzé, "Analysis and improvements of Adaptive Particle Refinement (APR) through CPU time, accuracy and robustness considerations", Journal of Computational Physics, Volume 354, 1 February 2018, Pages 552-575
L. Chiron, S. Marrone, A. Di Mascio, D. Le Touzé, "Coupled SPH–FV method with net vorticity and mass transfer", Journal of Computational Physics, Volume 364, 1 July 2018, Pages 111-136



Convergence of the smoothed particle hydrodynamics method for a specific barotropic fluid flow: Constructive kernel theory

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The SPH method is a popular numerical method for approximating the solution of complex fluid flow problems. Despite the fact that SPH was introduced quite some time ago and despite the fact that it has shown remarkable results in practical applications, the theoretical understanding of the method is still limited.

Earlier papers which deal with mathematical proofs of convergence are rare. There is the work by Ben Moussa and Vila, see for example [1], and the work by Di Lisio, Grenier, and Pulvirenti, see for example [3]. However, in both cases no full convergence results for the SPH method are given. Hence, these results are only of limited use in practical applications.

Since the SPH method is a kernel-based method, the employed kernel Φ will obviously play a crucial role in deriving a convergence result. In the case of the Euler equations with a barotropic equation of state, Oelschläger [4] stated kernel conditions and proved a convergence result for the SPH method using an energy-like error term. Unfortunately, this work was widely ignored within the SPH community due to its rather complicated conditions on the kernel and the fact that there were no known compactly supported kernels satisfying these conditions. Moreover, the result of Oelschläger was too weak to prove convergence of the particle trajectories.

In our paper, we were able to generalize and improve the results of Oelschläger to give convergence results for particle trajectories of the SPH method for the very first time. Moreover, we were able to rephrase the conditions on the kernel in a way which makes them actually verifiable. Finally, we were also able to construct a class of easy-to-calculate compactly supported kernels, which are based on the popular Wendland functions from [6] and which satisfy all required conditions. The paper contains a list of these new kernels.

For the convergence results, the kernel needs to satisfy the following conditions:

- The employed kernel Φ must be a convolution kernel, i.e. it must be of the form $\Phi=\Phi^r\Phi^r$ with a sufficiently smooth convolution root Φ^r.
- The convolution root Φ^r has to satisfy a moment condition of order m
- The convolution rootΦ^r has to satisfy an approximation condition of order L

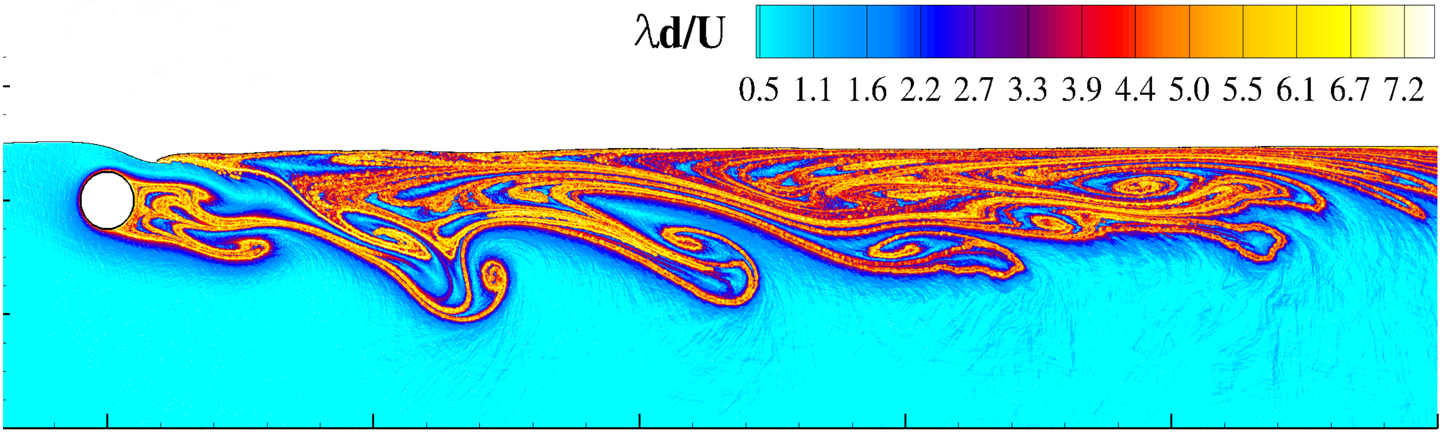
The first and the third condition were also stated in [4] by Oelschläger. The first condition implies particularly that the kernel Φ has a nonnegative, often even positive Fourier transform. Interestingly, Dehnen and Aly showed in [2] that a positive Fourier transform is favorable to avoid pairing instabilities. The moment condition has been used in earlier papers, see for example [5] by Raviart, to show convergence of particle methods for linear advection problems.

However, the combination of the moment condition and the approximation condition was the key element to prove stronger convergence results. This was done by first improving the convergence result of Oelschläger in the weak energy-like norm under certain assumptions on the smoothness of the root kernel Φ^r and the solution. These stronger results then allowed us to also conclude pointwise convergence of the particle trajectories. In all these cases, a specific, nonlinear relation between the smoothing parameter ε and the discretization parameter h is required.

We were also able to show that a simple extension of the Wendland functions satisfies the three required conditions. The Wendland function $\Phi_{d+2l,k}$, with smoothness parameter k and extended space dimension d+2l has, when used in the d-dimensional space, a convolution root $\Phi^r_{d+2l,k}$ that satisfies an approximation condition of order l-1. By taking a special linear combination of scaled versions of $\Phi_{d+2l,k}$ we can satisfy the moment condition for the convolution root of arbitrary order by preserving the approximation property for the linear combination. Since such functions are also radial functions represented by piecewise polynomials, they are easy to implement and efficient to calculate.

Early numerical tests of the new kernels show that in some situations they indeed show better results than the classical Wendland functions. Nonetheless, the proofs in this paper do not apply to the classical Wendland functions and, as they have shown good results in many applications, it remains an open problem, to mathematically prove that the SPH method also converges when these standard kernels are used.

[1] B. Ben Moussa and J. P. Vila, Convergence of SPH method for scalar nonlinear conservation laws, SIAM J. Numer. Anal., 37 (2000), pp. 863--887, https://doi.org/10.1137/S0036142996307119.
[2] W. Dehnen and H. Aly, Improving convergence in smoothed particle hydrodynamics simulations without pairing instability, Monthly Notices of the Royal Astronomical Society
[3] R. Di Lisio, E. Grenier, and M. Pulvirenti, The convergence of the SPH method, Comput.Math. Appl., 35 (1998), pp. 95–102.
[4] K. Oelschläger, On the connection between Hamiltonian many-particle systems and the hydrodynamical equations, Arch. Rational Mech. Anal., 115 (1991), pp. 297–310.
[5] P. A. Raviart, An Analysis of particle methods, in: Numerical Methods in Fluid Dynamics, Como, 1983, Lecture Notes in Math. 1127 (1985), pp. 243-324.
[6] H. Wendland, Piecewise polynomial, positive definite and compactly supported radial functions of minimal degree, Adv. Comput. Math., 4 (1995), pp. 389–396.



Interview with Andrea Colagrossi

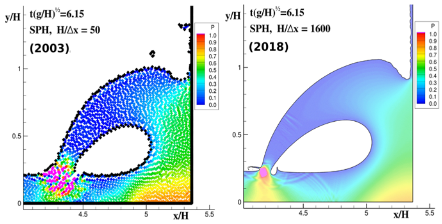

Dr. Andrea Colagrossi

Since 1998 Dr. Colagrossi he has been employed as researcher at CNR-INM (Institute for Marine Engineering of the Italian Research Council). In 2005 he obtained his PhD degree at the University of Rome 'La Sapienza'.
The main focus of his research is the theoretical and computational aspects of mesh-free numerical methods for fluid dynamics in naval and marine oriented research activities.
During the period 2006 - 2018 he has been a member of the Steering Committee of SPHERIC.

When (and why) did you decide to work in the field of research?
It happened gradually during my studies. Indeed, during the last years at university I started to select courses more oriented in applied mathematics and, in particular, the ones on fluid dynamics had attracted me for the richness of the theories involved. Finally, I did my master thesis on numerical fluid dynamics and I went to INSEAN. There, I started to develop a code based on potential flow theory for simulating the behaviour of ships in waves.
Incidentally the final mathematical model was a meshless method. Once I took the Bachelor degree, my supervisor Dr. Landrini was able to find contracts to complete this study at INSEAN and the story of my research activity started.

What is the coolest thing about your work?
Actually there is a number of "cool things": (i) I had the possibility of developing different codes applied to different problems. This forced me to study a wide range of interesting fluid dynamic models. (ii) The second point regards the collaboration that I had with more than thirty researchers. It was very interesting to work with so many friends. (iii) And finally, last but not the least, the work done with the Ph.D. students I supervised. The fact that you can help future generations of researchers is a great thing.
Being a researcher is a fantastic job, even if, often, many of us work in "unfair conditions". Seeing great young researchers struggling against this is not easy. But, we know, this is a very old story.

What motivated you to start working on SPH?
It happened just by chance: at the end of 1999 my supervisor Dr. Landrini



Enlarged view of a Dam break flow impacting against a vertical wall simulated with SPH. Left: results published on Colagrossi & Landrini (2003) using 5000 particles. Right: same simulation redone after fiftheen years with 5,120,000 particles (see Meringolo et. al 2019)

was involved in a research project funded by the Office of Naval Research for simulating breaking bow waves. For this project I went with him to the University of S.Barbara invited by Prof. Tulin. He had the great idea to use SPH for those problems. At that time I had just two papers on my desk: Simulating Free Surface Flows with SPH" by Prof. Monaghan and the one of MPS by Prof. Koshizuka. In the city of the "Beach-boys" and surfers, I began programming my first SPH code on a Pentium PC to simulate plunging breaking waves.

What question or challenge were you setting out to address when you started this work?

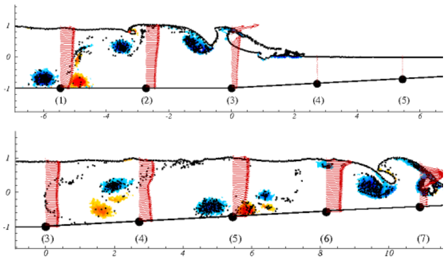
A few years ago the SPHERIC community had identified five Grand Challenges and during the recent years a lot of good answers to them have been published. Conversely, in 1999 every aspect concerning SPH was a Grand Challenge. For example, I spent a month to simulate a simple standing wave. I made the error of not spreading the particles on a cartesian lattice, to avoid a stepped profile of the curved free surface. I didn't know that this is something to avoid when dealing with the SPH !
On the other hand, during those years, there was a lot of space for developing new ideas while today the problems we face are more complex. We have plenty of papers on SPH each year but new ideas are few becouse they require more and more efforts.
However, coming back when I started to work with SPH, another main problem was the difficulties in publishing papers since referees did not trust on a new numerical method mainly based on a great physical intuitions rather than a well-founded mathematical theory.
Reviewer #3: How do you enforce the dynamic boundary condition on the free-surface ?...
Author: uhhhm... in reality I did not do anything but it seems to work, why? I do not know!

What are the biggest scientific achievements you have reached during your career?

The best results are probably the one published on my first JCP paper: (i) to have understood that the "classical" pressure term of the SPH was not suitable when dealing with two fluids with large density ratio and (ii) that the density field needs to be filtered in some way in order to remove the spurious numerical high frequencies. Then, thanks to the collaborations with other researchers, we end up with the delta-SPH scheme introducing directly the dissipation term in the continuity equation. The 2003 JCP paper was my first publication on a high quality journal and it reaches today more than one thousand citations. I do not think I will be able to repeat such a target with an another work. I have to thank my supervisor Maurizio Landrini who helped me in writing this paper. Unfortunately in the same year he died only 40 years old in a tragic accident. During 2002 Maurizio and Prof. Ferrant organized a student exhange program between ECN and INSEAN. Thanks to this a strong and successful collaboration between the two institutes on the SPH topic began.

In your opinion what are the most relevant challenges that the SPHERIC community should address in the near future?

The creation of the SPHERIC community was a great idea: give the opportunity to share ideas between different european research groups. Europe had a great role in the development of the SPH method, from the collaboration of Prof. Gingold and Prof. Monaghan in Cambridge up to the



SPH simulation of a breaking bore climbing an inclined beach (3°): vorticity contour levels in the flow field and vector velocity profiles at the fixed vertical sections. The small dots are the particles which belong to the free surface at initial time. These particles have been trapped into the fluid during the breaking processes (see Landrini et al. 2007). The simulation has been performed at S. Barbara in 2002.

development of different research groups and consortia for handling big codes.
All of this give us great steps forward for the SPH model: new techniques for handling solid surfaces, the particle shifting technique for increasing the accuracy, the surface tension models, the adaptive particle resolution algorithms, coding on the GPUs, and so on.
The network of recent years is even larger thanks also to the many collaborations born between Europe and Asia. The relevant challenges for this community in the near future are to support this network in order attract funds for student exchange programs and for projects where academical and industrial partners can work together.
In the beginning of the millenium we were a small community that inherited the SPH method from the astrophysical field to extend it into a more general fluid dynamic context. It is very difficult to predict if and in which way the SPHERIC community will evolve, it largely depends on the fields where this method can be successful applied, as stressed by the SPHERIC chairman (B. Rogers) in the last workshop. Today, for sure, we can say that, also thanks to us, Particle Methods have well established their positions in the "Olympus" of the numerical methods.

What is your vision of the current state of the art of SPH?
I think we are still in the ascendent part of the life-curve of this method. Thanks to the development of the recent years important drawbacks have been removed. The are still advantages of this method with respect to others, like the ability in accurately simulating flow characterized by interfaces subjected to large motion. At the same time, also mesh-based methods are evolving, in particular the Adaptive Mesh Refinement (AMR) technique allows to get very good results in problems where SPH has been successfully used, i.e. water impacts, sloshing flows, breaking waves, multi-phase flows, etc.
Mesh-based methods with AMR will be the main competitors of the SPH. Furthermore, I think that the cross-validation between mesh-based and particle methods can be an advantages for both the methodologies. This was the case in my personal experience, since experimental data are not always so helpful when you need to check novel numerical models.

What is your vision of SPH in 5/10 years?
A crucial aspect of the future developments of the SPH is mainly linked to the possibility to have open-source algorithms in order to manage the complexities of the advanced SPH codes available today. Codes that are written for GPUs or multiple CPUs with adaptive particle resolutions and contain many possible choices regarding different SPH models, different particles-interaction models, different time integrators, and so on.
I belong to a generation where numerical codes were programmed from scratch, I programmed by myself even the linked-list cell. Today it is not more the case, the algorithms reach a too high level of complexity and you cannot spend time to repeat what other peoples (most likely with informatic knowledge higher than yours) did before you. On the other hand building a code using multiple pieces, which are essentially black boxes, requires a special attention and can lead to constraints that can limit future developments and possible new research paths.
Another important role will be played by the coupling techniques, the possibility to use SPH combined with other methods will possibly open new frontiers improving CFD codes through the right combinations of different models.

Using DualSPHysics to assist in the design of a complex model basin beach

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Edinburgh Designs has been building wave test tanks for the last 30 years, and are now building a large research installation in Singapore for the testing of scale models of offshore structures and ships.

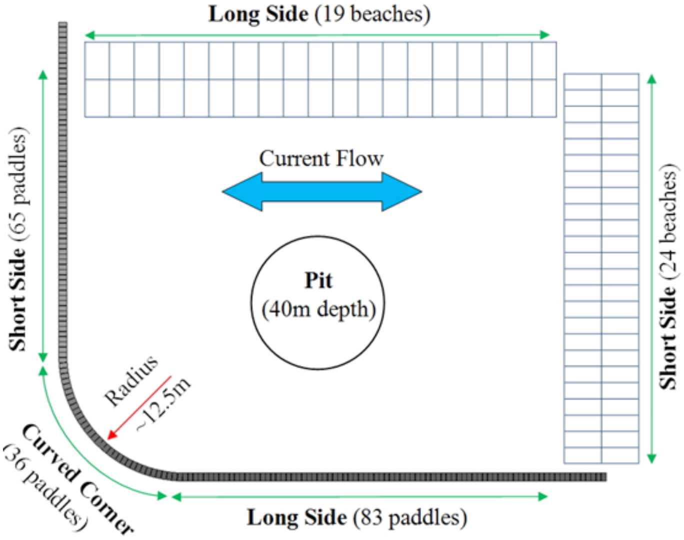

Figure 1 - General arrangement of Singapore tank

The basin is 48m wide, 60m long and has a moveable floor with a maximum working water depth of 12m as shown in Figure 1. Wavemakers are to be installed on two adjacent sides and beaches on the other two. The basin also has current that flows through the short side beach, and the long side beach can be reconfigured as a wall.
A critical part of any deep-water wave basin is that the beach absorbs as much of the incident wave as possible. Beaches rely on non-linear effects, either by breaking the wave on a sloping surface or by dissipating the wave energy in absorbing material.
A hybrid beach design was selected for the Singapore basin, using a lattice of concrete beams on a sloping surface. This design has been used in the US Navy's MASK research basin and Figure 2 shows the tank testing carried out at the University of Minnesota [1] to optimise the beach.
The upper section of the Singapore beaches was tested in our wave tank in Edinburgh at a scale of approximately 3:1. Sine and spectral waves were used to measure the reflection from the beach, using a linear array of gauges.
By directing parallel waves onto the beach, the incident and reflected waves can be separated. This was originally achieved by measuring the sine wave amplitudes at the node and antinode of the wave.
Waves are now measured with multiple probes and the results analysed. We use the least squares method as proposed by Mansard and Funke [3]. Figure 3 shows the results we obtained for different amplitude PM spectral waves, demonstrating good performance over the critical frequency range.
With this stage complete, the upper sections of the beach were designed and installed as shown in Figu-

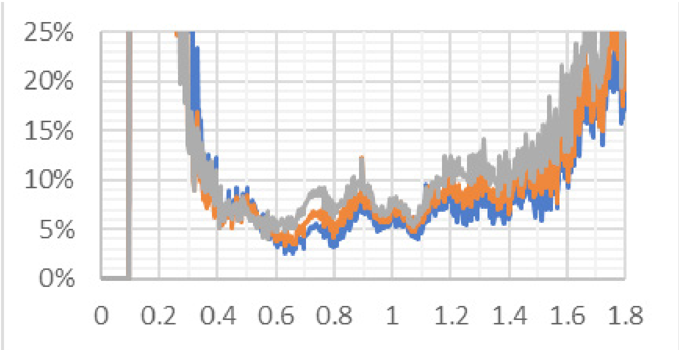

Figure 3 - Performance of scale beach in test tank

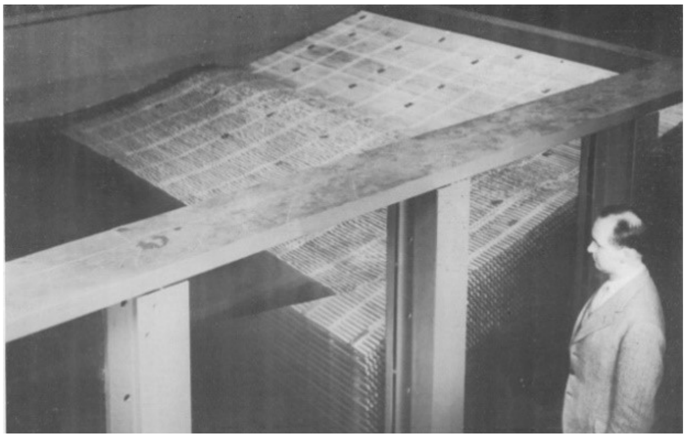

Figure 2: University of Minnesota test tank

Figure 4: Upper beach installed in Singapore

re 4. Now we had a problem with how to design the lower parts of the beach. We used OpenFoam for designing the flow in the tank and considered using the free surface solver for this task. At this time, we also became aware of DualSPHysics, that not only appeared to handle the free surface better, but also avoided the need for meshing.
We downloaded the SPH code and associated examples. However, it was immediately apparent that we would need to run the CUDA code on a GPU. We found that Tesla K80 cards, each with two GPU processors, could be bought on E-bay. They just need a fan lashed on to keep them cool!
Alex Chow came from Manchester to help us understand the settings and we were soon generating our first results. For this work we built a computer with two K80 cards on a dual processor mother board.

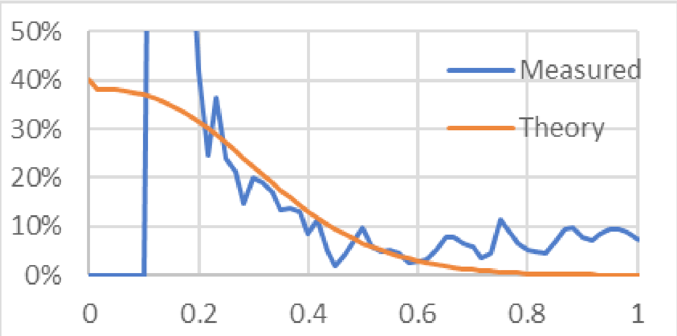

Figure 6: Reflection results for step change in depth

The depth of the front edge of the beach is important in determining the lowest frequency wave that can be absorbed. Marshall & Nagdi proposed in their paper [2] that the reflection at a step change in depth, could be predicted by a simple formula based on the wavenumber at the two depths.
To test this prediction, a three second PM spectrum was generated within a 2D model. This was directed

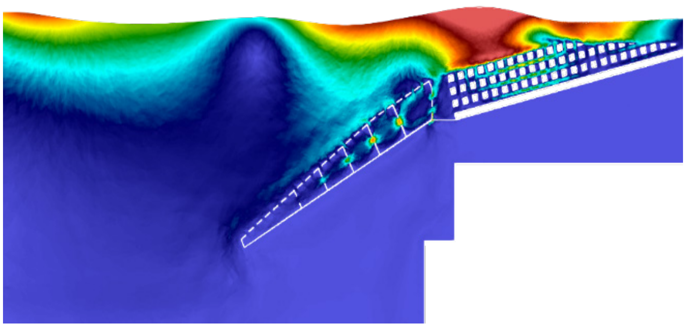

Figure 7: Long side beach with 3s PM wave

at a step change from 6m to 1.2m water depth. The water height at multiple locations was measured and the resultant reflection generated as for a conventional wave tank.
The results in Figure 6 show a good correlation between theory and simulated results over the central frequencies of the generated wave spectrum. This result not only gave us confidence in the step change equation, but also in the capability of DualSPHysics to provide results that we could utilise for our work.

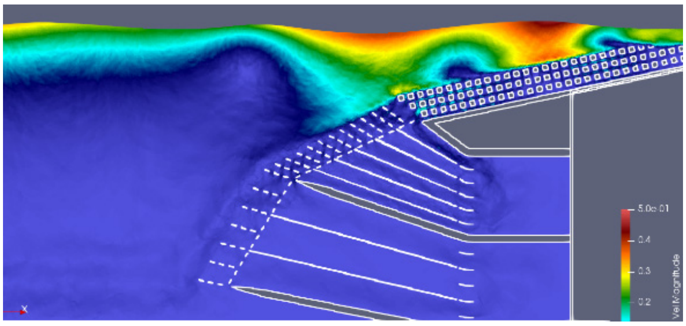

Figure 8: Final short side beach design

The lower section of the long side beach can be lowered to form a beach or raised to form a wall. This movable section is to be made with perforated steel plates on its upper surface, which need to match the impedance of the concrete slats to prevent reflection at the interface.
A variety of designs and porosities were tested and the final design is shown in Figure 7. To generate the results for this work required the use of all four GPUs for about two months.
The lower section of the short side beach must allow flow to pass through, so that waves and current can be combined. Although we had predicted that the waves could be blocked by dividing up the flow ducts with plates, the number and length of these ducts was not known. As for the long side, several designs were tested and the results compared. The optimised design is shown in Figure 8.
DualSPHysics has proved capable of supporting detailed beach design. We look forward to using improvements in the code to resolve known issues in the existing code base. We are also hopeful that increases in performance will allow us to work with smaller particle size

[1] LG Straub, CE Bowers & JB Herbich (1957), Project report No 54, St Anthony Falls Hydraulic Laboratory, University of Minnesota:
[2] J.S. Marshall and P.M. Naghdi. (1990), Wave Reflection and Transmission by Steps and Rectangular Obstacles in Channels of Finite Depth. Theoretical and Computational Fluid Dynamics 1: 287-301
[3] Mansard, E.P.D., Funke, E.R. (1980), The measurement of incident and reflected spectra using a least squares method. Proc. 17th Costal Eng. Conf., ASCE, pp. 154 – 172.

4th DualSPHysics Users Workshop

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The 4th DualSPHysics Users Workshop was hosted at Instituto Superior Técnico, University of Lisbon, Portugal, on the 22nd to 24th of October 2018. DualSPHysics is an open-source SPH code jointly developed by the Universidade de Vigo (Spain), the University of Manchester (UK), the University of Parma (Italy), University of Lisbon (Portugal) and Flanders Hydraulics (Belgium). The software package (www.dual.sphysics.org) is free, open-source and leverages the computing power of graphics processing units (GPUs) to put SPH in the hands of engineers and researchers; it has been downloaded thousands of times since its release in 2011 (Crespo et al., 2015). Following the success of the three Users Workshops in 2015, 2016 and 2017, users and developers of the DualSPHysics code showed how they have been applying and modifying the code.

The first day was devoted to a hands-on session, exploring the toolchain from pre-processing, solving and post-processing the results. The graphical user interface (http://design.sphysics.org/) updates were introduced, as well as the basic functionalities of the solver.

On day two Professors Ramón Gomez-Gesteira (University of Vigo) and Benedict Rogers (University of Manchester) described the trajectory of DualSPHysics, and its current and future challenges. Dr. Alexandro Crespo (University of Vigo) introduced the novelties of beta V4.3 along the toolchain and provided the attendees with access to the download page. Orlando Garcia-Feal (University of Vigo) closed the morning by giving a grand tour of the architecture of the code and performing a live implementation of a heat equation feature, both on the CPU and GPU solvers. The afternoon had four sessions of brief talks, where industrial and academic users presented their work and developments around DualSPHysics.

Day three was devoted to keynotes on the most relevant novelties released or updated in V4.3: Dr Corrado Altomare (University of Gent) detailed the modelling of sea waves and states; Dr Georgios Fourtakas (University of Manchester) showed the released work on multiphase simulation; Dr Angelo Tafuni (New Jersey Institute of Technology) released the much anticipated open Inlet/outlet boundary conditions; and Dr Ricardo Canelas showcased the the DualSPHysics-Chrono implementation.

Closing the workshop, an open-floor debate was held on questions regarding the software implementation and ways to contribute to the project.

More information and the presentations given at the workshop are available at the workshop site http://dual.sphysics.org/4thusersworkshop/.

http://dual.sphysics.org/4thusersworkshop/

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